Meta-Analytics: QUBO Based Portfolio Model

QUBO and QUBO-Plus Modeling Note
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Abstract

The importance of portfolio models and their practical applications is becoming increasingly highlighted by recent developments, including the recognition that they can be approached by quantum computing through the quadratic unconstrained binary optimization (QUBO) model. This brief note shows how a classical QUBO formulation for portfolio optimization can be enhanced by a QUBO-Plus formulation which incorporates a budget constraint and can be solved highly efficiently by latest advances in QUBO solution technology.

Background

Portfolio planning embraces a wide range of important applications in the field of practical optimization. The challenge of selecting a portfolio of assets to maximize return, accounting for risk and diversity and correlated behavior, and including subjective evaluations where applicable, appears in multiple settings and offers significant reward if solved effectively.

A new dimension for approaching these applications has come with the emergence of quantum computing, and the discovery that the QUBO model at the focus of numerous quantum computing initiatives can embrace portfolio applications. Breakthroughs in quantum hardware promise to elevate quantum computing to a pre-eminent position for solving optimization problems as the field matures over the next five to ten years. Fortunately, it is unnecessary to wait for quantum computing to reach maturity before gaining the advantage of dramatic improvements in our ability to solve key optimization problems. The realm of Quantum Bridge Analytics ([1] and [2]) is fueling advances that provide unprecedented gains in solving optimization problems today as a foundation for transitioning to the quantum computing environments of tomorrow.

Motivated by these developments, this note describes the form of a QUBO model that enables Quantum Bridge Analytics to be applied to selecting asset portfolios, and to bring portfolio optimization into the forefront of applications that can benefit from these new advances.

Basic Model

We start from the quantum annealing and portfolio analysis model of [3], which is considered the fundamental approach for representing a portfolio problem as a QUBO model. The problem consists of forming a portfolio from a set of n assets with known attributes such as asset returns and pairwise correlations, to give rise to a portfolio model of the form:

$$\min f(x) = \sum_{i=1}^{n} q_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} q_{ij} x_i x_j$$

where

$x_i = 1$ if asset i is selected, otherwise 0.
q_i = a measure of return.
q_{ij} = a measure of diversity associated with assets i and j.

For a particular portfolio application, numerical values for the model parameters are derived from raw asset data as follows:

- From raw return data, a risk adjusted return value (perhaps a Sharpe ratio) is computed.
- These values are put into equally spaced groupings, ranked and mapped into a sequence of integer values to be taken as the q_i values.
- Likewise, the asset correlations values are put into groups, with each group having an associated integer to be used as the q_{ij} values.

This mapping process allows a great deal of opportunity to fold subjective and other desirable influences into the selection process.

Note that the model above is in the form of a QUBO model: $\min x^t Q x$, where Q is the n by n matrix in upper triangular form whose diagonal coefficients are given by the q_i values and whose coefficients above the diagonal are given by the q_{ij} values.

Enhanced Model

Drawing on the ideas in [1], we now modify this starting model to accommodate a pre-set portfolio size by imposing a cardinality constraint and/or a budget limit by imposing a knapsack constraint.

Let

size = the desired portfolio size (number of assets to be chosen)

c_j = cost of asset j

b = budget limit

Then the enhanced model becomes

$$\min f(x) = \sum_{i=1}^{n} q_i x_i + \sum_{i=1}^{n} \sum_{j=i+1}^{n} q_{ij} x_i x_j$$

$$\sum_{j=1}^{n} c_j x_j \le b \text{ (budget constraint)}$$

$$\sum_{j=1}^{n} x_j = size \text{ (portfolio size restriction)}$$

This constrained problem formulation has the advantage of being able to model much more general and practical portfolio problems. We can additionally incorporate budget constraints for different sets of investments in the Enhanced Model and include upper and lower bounds on the number of investments instead of choosing an exact number. Utilizing the Quantum Bridge Analytics perspective of [1] and [2], the resulting formulation can be expressed as a pure QUBO model or solved directly as a QUBO-Plus model.

Recent computational advances in solving QUBO and QUBO-Plus models reported in [4] make it possible to solve these portfolio optimization applications with unprecedented effectiveness, obtaining better solutions than previously accessible and solving problems much larger than handled in the past.

References

[1] F. Glover, G. Kochenberger and Y. Du (2019) "Quantum Bridge Analytics I: A Tutorial on Formulating and Using QUBO Models," 4OR

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[2] F. Glover, G. Kochenberger, M. Ma and Y. Du (2020) "Quantum Bridge Analytics II: Combinatorial Chaining for Asset Exchange," Cornell University Library, arXiv: http://arxiv.org/abs/1911.03036

[3] D. Venturelli and A. Kondratyev (2018) "Reverse Quantum Annealing Approach to Portfolio Optimization Problems," Cornell University Library, arXiv 1810.08584v2 [quant-ph]

[4] F. Glover, G. Kochenberger and Y. Du (2020) "New Advances and Applications for Solving the QUBO Problem," invited chapter in The Quadratic Unconstrained Binary Optimization Problem, A. Punnen, editor, Springer (to appear in 2021).